Development of Constitutive Equation on Superplastic RS P/M Mg-Y-Zn Alloy

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A constitutive equation of rapid solidification (RS) powder metallurgy (P/M) magnesium alloy was constructed analytically from normalized strain rate-stress plots. Data related to RS P/M magnesium alloy showed deviation from those of conventional superplastic magnesium alloys; they indicated greater strength or lower strain rates than conventional superplastic magnesium alloys because the grains have a unique structure on superplastic flow. A constitutive equation for superplastic flow was developed using a metal matrix composite model.

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1. Introduction

Magnesium alloys are attracting great attention for their ecological benefits because magnesium alloys are the lightest of all structural alloys. Recently, it has been reported that the addition of rare earth (RE) elements remarkably improves magnesium's mechanical properties at room temperature and high temperatures.^{1–5)} Kawamura *et al.* have reported that the powder metallurgy (P/M) magnesium alloy proposed by rapid solidification (RS) offers excellent mechanical properties by the addition of yttrium atom.^{6,7)} The resultant chemical composition became Mg–2 at%Y–1 at%Zn with initial grain size of ca. 200 nm. This RS P/M magnesium alloy had very high yield strength above 600 MPa at room temperature and high creep resistance. The RS P/M magnesium alloy also exhibited superplastic behavior at 623 K and at strain rates of 10^{-2} to $10^0 \text{ s}^{-1.6}$

Superplasticity describes the ability of a crystalline material to exhibit large strains when pulled in tension. Superplastic forming is a viable technique to fabricate hard-to-form complex shapes.⁸⁾ On the other hand, this forming phenomenon is usually attained at low strain rates of 10^{-3} s⁻¹ and at temperatures of ca. 0.8 T_m , where T_m is the melting point of the materials. The superplastic strain rate range is rather low for conventional forming of structural materials. For that reason, the commercial viability of superplastic materials is limited. To resolve the problems described above, recent experimental evidence suggests that a grain-size reduction will increase the strain rate and/or decrease the temperature for optimum superplastic flow.^{8–10)}

In RS P/M magnesium alloy, high-strain-rate superplasticity, which is defined as superplasticity occurring at or above 10^{-2} s⁻¹ [JIS H7007],¹¹⁾ has been obtained,⁶⁾ because of its ultra fine grains of ca. 200 nm. Such high-strain-rate superplasticity reduces energy costs and improves production rates. In the future, it is expected that the RS P/M magnesium alloy will become increasingly applicable to production of structural components; it is garnering attention for use as a next generation material. However, precedent studies have indicated only superplasticity without the deformation mechanism.⁶⁾ To apply this material to actual superplastic forming processes, it is important to understand its superplastic behavior and develop a constitutive equation for superplastic flow on RS P/M magnesium alloy. Therefore, this study analyzed the deformation mechanism and developed a constitutive equation.

2. Analysis of Superplastic Properties Flow on RS P/M Magnesium Alloy

The constitutive equation to describe superplastic flow in a pseudo single-phase alloy can be expressed generally as^{12}

$$\dot{\varepsilon} = A \left(\frac{Gb}{kT}\right) \left(\frac{\sigma - \sigma_0}{G}\right)^n \left(\frac{b}{d}\right)^p D \tag{1}$$

where $\dot{\varepsilon}$ is the matrix strain rate, A is a constant, k is Boltzmann's constant, T is the temperature, G is the shear modulus, b is Burger's vector, d is the grain size, σ is the flow stress, σ_0 is the threshold stress, n is the stress exponent (n = 1/m: m is the strain rate sensitivity exponent), p is the grain size exponent, and D is the diffusion coefficient. Sherby and Wadsworth suggested an effective diffusion coefficient, D_{eff} , for analyses of superplastic flow.¹³⁾ The effective diffusion coefficient is described as a combination of the grain boundary diffusion coefficient, D_{gb} , and the lattice diffusion coefficient, D_{L} , as¹³⁾

$$D_{\rm eff} = D_{\rm L} + x \left(\frac{\pi}{d}\right) \delta D_{\rm gb} \tag{2}$$

where δ is the grain boundary width approximated as 2b and x is an unknown constant. Recently, the x term was estimated as 1.7×10^{-2} for superplasticity.¹⁴⁾ The constitutive equation for superplastic flow, as derived from Eqs. (1) and (2), is expressed as^{14,15)}

$$\dot{\varepsilon} = 1.8 \times 10^6 \left(\frac{Gb}{kT}\right) \left(\frac{\sigma - \sigma_0}{G}\right)^2 \left(\frac{b}{d}\right)^2 D_{\text{eff}}$$
(3)

Variation in $(\dot{\epsilon}/D_{\rm eff})(kT/Gb)(d/b)^2$ as a function of $(\sigma - \sigma_0)/G$ is plotted in Fig. 1, where $D_{\rm eff}$ is assumed as $[D_{\rm L} + (1.7 \times 10^{-2})(\pi \delta/d)D_{\rm gb}]$ for conventional superplastic magnesium alloys¹⁴ and the RS P/M magnesium alloy.⁶ The solid line and open symbols respectively indicate results for

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Fig. 1 Variation in $(\dot{\epsilon}/D_{\rm eff})(kT/Gb)(d/b)^2$ as a function of $(\sigma - \sigma_0)/G$, where $D_{\rm eff}$ is taken as $[D_{\rm L} + (1.7 \times 10^{-2})(\pi \delta/d)D_{\rm gb}]$, for conventional superplastic magnesium alloys and RS P/M magnesium alloy.

summarized conventional superplastic magnesium alloys and data of the RS P/M magnesium alloy. Superplastic flow in the RS P/M magnesium alloy is represented as a single straight line with a slope of 2 in these normalized plots. On the other hand, the flow stress of RS P/M magnesium alloy is about two orders of magnitude higher than that of conventional superplastic magnesium alloys because of strengthening. Therefore, it is apparent that the conventional constitutive equation in eq. (3) is inapplicable to that for RS P/M magnesium alloy. The constitutive equation and the origin of the slight deviation for RS P/M magnesium alloy are discussed in subsequent sections.

3. Development of a Constitutive Equation for Superplastic Flow on RS P/M Magnesium Alloy

Previous detailed microstructural observations have indicated that the microstructure in RS P/M magnesium alloy is composed of grain sizes of ca. 200 nm.¹⁶⁾ These grains are classifiable into two types in terms of structure: hcpmagnesium solid solution grain and fine lamellar grain with a novel long-period ordered structure. The volume fraction of these lamellar grains is roughly estimated to be up to 40%. It has been considered that lamellar grains affect the strengthening of RS P/M magnesium alloy. In this analysis, this lamellar grain is a component of a strengthening colony, such as in reinforcing particles in a metal matrix composite, which affects the superplastic flow.

Several researchers have pointed out that these particles' effects would be marked, and that superplastic flow would be altered from conventional superplasticity at high strain rates.^{12,17–20} Mabuchi and Higashi have proposed that the

constitutive equation for superplastic flow on the metal matrix composite includes parameters such as volume fraction, size and spacing of reinforcing particles. It is given as^{19}

$$\dot{\varepsilon} = B \left(\frac{Gb}{kT}\right) \left(\frac{\sigma - \sigma_0}{aG}\right)^2 \left(\frac{b}{d}\right)^2 \left(\frac{\lambda}{b}\right) D \tag{4}$$

where *B* is the constant, λ is the particle spacing, and *a* is the strengthening coefficient, which is approximated from the calculation of the perfectly elastic plastic law as²¹⁾

$$a = 1 + 2(2+c)V_{\rm f}^{3/2} \tag{5}$$

where *c* is the aspect ratio of particle and $V_{\rm f}$ is the volume fraction of particle. The value of λ is also given as²²⁾

$$\lambda = \frac{d_{\rm p}}{2} \sqrt{\frac{3\pi}{2V_{\rm f}}} \tag{6}$$

where d_p is the particle size. They obtained a constant value of *B* in eq. (4) equal to 6.5×10^{-1} using data for aluminum matrix composites.¹⁹

Using eqs. (4)–(6), variation in $(\dot{\epsilon}/D_{\rm eff})(kT/Gb)(d/b)^2$ - (λ/b) as a function of $(\sigma - \sigma_0)/aG$ is plotted in Fig. 2, where the reinforcing particle replaced the strengthening colony having equi-axed grains of ca. 200 nm, in this study. This figure includes the conventional metal matrix composite¹⁹⁾ and the various volume fraction of strengthening colony. Figure 2 shows that plotted data on RS P/M magnesium alloy fitted with the solid line in the volume fraction of strengthening colony having 0.4. In addition, a volume fraction equal to 0.4 agrees with previous microstructural investigations.¹⁶⁾ Therefore, the constitutive equation for



Fig. 2 Variation in $(\dot{\epsilon}/D_{\rm eff})(kT/Gb)(d/b)^2(\lambda/b)$ as a function of $(\sigma - \sigma_0)/aG$ on Mg–Y–Zn alloy with various volume fractions.

superplastic flow on RS P/M magnesium alloy can be developed by regarding the strengthening colony as a reinforcing particle.

4. Influence of Reinforcing Colonies on Superplastic Behavior on RS P/M Magnesium Alloy

As described in the previous section, a slight deviation from conventional superplastic magnesium alloys is caused by the existence of the strengthening colonies. The constitutive equation was developed considering the influence. It is also important to elucidate the influence of those colonies on superplastic flow.

In general, two possible locations exist for the particles: at the grain boundary and at the grain interior. When particles are located at the grain boundaries, the effects of intergranular particles are apparent from the viewpoint of stress relaxation around the particles by diffusional flow. On the other hand, when particles are located in the grain interior, they hinder the movement of dislocation from one side of the grain to the other. Watanabe *et al.* proposed that a critical strain rate pertains whether or not the intergranular or intragranular particles affect the slip accommodation process on superplastic flow.²⁰⁾ Critical strain rates of intergranular particles, $\dot{\varepsilon}_{c1}$, and intragranular particles, $\dot{\varepsilon}_{c2}$, are given as²⁰⁾ the following.

$$\dot{\varepsilon}_{c1} = 6.6 \times 10^{-7} \left(\frac{Gb}{kT}\right) \left(\frac{1}{b^4}\right) \\ \times \left\{\frac{\Omega(D_{\rm L} + 5\delta D_{\rm gb}/d_{\rm p})}{d_{\rm p}}\right\}^2 \left(\frac{1}{V_{\rm f}^2}\right) \frac{1}{D_{\rm eff}} \qquad (7a)$$
$$\dot{\varepsilon}_{c2} = 9.24 \left(\frac{\Omega}{kT}\right) \left(\frac{b}{d}\right) \left(\frac{1}{d_{\rm p}^2}\right) D_{\rm L}$$

$$\times \frac{GG^*(1+v^*)}{\{G^*(1+v^*)+2G(1-2v^*)\}}$$
(7b)

In those equations, Ω is the atomic volume, G^* is the shear modulus of the particle, and v^* is the Poisson ratio of the particle. When $\dot{\varepsilon} < \dot{\varepsilon}_{c1}$, the stress concentrations around reinforcements are relaxed only by diffusional flow. The deformation mechanism of superplasticity is the same as that for unreinforced metals. On the other hand, when $\dot{\varepsilon} \geq \dot{\varepsilon}_{c1}$, a special accommodation process by an accommodation helper, such as a liquid phase, is required to relax the stress concentration around the particles. In addition, when $\dot{\varepsilon} \leq \dot{\varepsilon}_{c2}$, intergranular particles are in a fully relaxed state; consequently, the particles do not affect the slip accommodation process. However, when $\dot{\varepsilon} > \dot{\varepsilon}_{c2}$, the dislocations are suggested to be piled up at the particles because the gliding dislocations cannot bypass the particles. The pile-up at the particle is analogous to a dislocation pile-up that occurs at grain boundaries.

Based on this notion, the variation in the critical strain rates, $\dot{\varepsilon}_{c1}$ and $\dot{\varepsilon}_{c2}$, are shown respectively as a function of temperature in Fig. 3(a) for intergranular particles and Fig. 3(b) for intragranular particles. Table 1 lists the material characteristics.²³⁾ The superplastic regions in the RS P/M magnesium alloy are indicated as open symbols. Figure 3(a) shows that the critical strain rates for intergranular particles,



Fig. 3 Variation in critical strain rate as a function of temperature on the RS P/M magnesium alloy in (a) an intergranular particle and (b) an intragranular particle.

Table 1 List of material factors of magnesium.²³⁾

Materials factor	
Atomic volume, Ω/m^3	2.33×10^{-28}
Burger's vector, b/m	3.21×10^{-10}
Grain boundary width, δ/m	6.42×10^{-10}
Shear modulus, G/MPa	$1.66\times 10^4 \{1-0.49(T-300)/924\}$
Grain boundary diffusion coefficient, D_{gb}	$7.8 \times 10^{-3} \exp(-Q/RT)$
Activation energy, $Q/kJmol^{-1}$	92
Lattice diffusion coefficient, $D_{\rm L}$	$1.0 \times 10^{-4} \exp(-Q/RT)$
Activation energy, $Q/kJmol^{-1}$	135

The shear modulus, G^* , and Poisson's ratio, v^* , of the reinforcing colony are unknown. For that reason, v^* is presumed as 0.3, and G^* is the value of magnesium given in Table 1.

 $\dot{\varepsilon}_{c1}$, are fundamentally faster than that of the experimental superplastic region: the intergranular particles do not affect the superplastic flow. The present analysis suggests that the strengthening in RS P/M magnesium alloy is not associated with the presence of strengthening colonies. On the other hand, from Fig. 3(b), it is apparent that the critical strain rates for intragranular particles, $\dot{\varepsilon}_{c2}$, are lower than those of the experimental superplastic regions. The diffusional relaxation around a strengthening colony is not completed during the slip accommodation process. In this case, dislocations are suggested to be piled up not only at grain boundaries, but also at the strengthening colony, because the gliding dislocations cannot bypass the strengthening colony. It has been proposed that the dislocation pile-up at the colony would contribute to strengthening because the pile-up at the colony is analogous

to the case of dislocation pile-up at grain boundaries at ambient temperature. However, future studies must investigate the mechanical properties of these strengthening colonies in greater detail.

5. Summary

Normalized plots of the magnesium alloys show that the flow stresses on RS P/M magnesium alloy are much greater than those on conventional superplastic magnesium alloys. These higher flow stresses result from particle strengthening; these reinforcing colonies have a unique structure. The constitutive equation for superplastic flow on RS P/M magnesium alloy was constructed using a conventional metal matrix composite model.

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